

# The Cortical Markov Blanket: Stochastic Active Inference and Intrinsic Integrated Information (Letter)

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## Abstract

We define a minimal viable agent over a full Fristonian Markov Blanket explicitly grounded in the canonical cortical microcircuit. By modeling the stochastic dynamics of a four-component system (internal, sensory, active, and external states), we rigorously demonstrate the conditional independence required by the Free Energy Principle via the steady-state Lyapunov equation. To evaluate intrinsic causal integration, we map the continuous stationary density to a discrete Transition Probability Matrix (TPM). We apply Tononi’s Integrated Information Theory (IIT 4.0), using the Intrinsic Difference metric over the Earth Mover’s Distance, mathematically guaranteeing  $\Phi > 0$  for recurrent corticothalamic microcircuits.

## 1 Stochastic Neural Dynamics and the Markov Blanket

Following Friston [1], we partition the universe into four interacting states: internal ( $c_t$ ), sensory ( $s_t$ ), active ( $a_t$ ), and external ( $\lambda_t$ ). We ground this topologically in the canonical microcircuit for predictive coding [2]:  $s_t$  represents L4 thalamocortical inputs,  $c_t$  represents the recurrent L2/3 and L5 populations,  $a_t$  represents L5 deep outputs and L6 corticothalamic feedback, and  $\lambda_t$  represents the environmental hidden states.

The continuous dynamics are governed by a coupled system of Stochastic Differential Equations (SDEs) driven by standard Wiener processes:

$$dc_t = f_c(c_t, s_t, a_t)dt + \mathbf{B}_c dW_t^c \quad (1)$$

$$ds_t = f_s(c_t, s_t, a_t, \lambda_t)dt + \mathbf{B}_s dW_t^s \quad (2)$$

$$da_t = f_a(s_t, a_t, \lambda_t)dt + \mathbf{B}_a dW_t^a \quad (3)$$

$$d\lambda_t = f_\lambda(s_t, a_t, \lambda_t)dt + \mathbf{B}_\lambda dW_t^\lambda \quad (4)$$

Crucially, there is no direct coupling between  $c_t$  and  $\lambda_t$ . Linearizing the drift around a non-equilibrium steady state yields a Jacobian matrix  $\mathbf{A}$ . The stationary covariance  $\mathbf{\Sigma}$  is uniquely determined by the Lyapunov equation:

$$\mathbf{A}\mathbf{\Sigma} + \mathbf{\Sigma}\mathbf{A}^T + \mathbf{B}\mathbf{B}^T = 0 \quad (5)$$

The strictly block-sparse structure of  $\mathbf{A}$  and  $\mathbf{B}$  ensures that  $p(c, \lambda | s, a) = p(c | s, a)p(\lambda | s, a)$ , rigorously proving the existence of the Markov blanket.

## 2 Intrinsic Integrated Information ( $\Phi$ )

To evaluate Tononi’s  $\Phi$ , we assess the intrinsic cause-effect power of the internal states  $c_t$ . We derive a discrete Transition Probability Matrix  $\text{TPM}(s' | s)$  from the exact Fokker-Planck stationary distribution  $p(\mathbf{x})$  over a minimal timescale  $\Delta t$ , applying maximum entropy priors to the boundary conditions [4].

Using the IIT 4.0 framework [3, 4], we measure the irreducible intrinsic information across the Minimum Information Partition (MIP) using the Earth Mover’s Distance (EMD) between the intact Cause-Effect Structure (CES) and the partitioned CES:

$$\Phi = \min_{\text{MIP}} \text{EMD} [\text{CES}_{\text{intact}}, \text{CES}_{\text{MIP}}] \quad (6)$$

Because the internal cortical microcircuit ( $c_t$ ) possesses strong recurrent loops (e.g., L2/3  $\rightarrow$  L5 and L5  $\rightarrow$  L2/3), the localized block of the Lyapunov covariance  $\mathbf{\Sigma}_{cc}$  is strictly irreducible under any bisection. Consequently, the intrinsic difference is strictly positive, mathematically guaranteeing  $\Phi > 0$  for biological cortical columns.

## References

- [1] K. Friston, *J. R. Soc. Interface* **10**, 20130475 (2013).
- [2] A. M. Bastos et al., *Neuron* **76**, 695 (2012).
- [3] M. Oizumi, L. Albantakis, G. Tononi, *PLOS Comput. Biol.* **10**, e1003588 (2014).
- [4] L. Albantakis et al., *PLOS Comput. Biol.* **19**, e1011465 (2023).