The Thermodynamic Bias Toward Manifolds in Causal Sets: Mean-Field Prerequisites for Lorentz Invariance (Letter)

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June 1, 2026

Abstract

The extraction of the Minkowski metric from discrete causal graphs is complicated by the Kleitman-Rothschild (KR) order collapse. We introduce a thermodynamic partition function governed by the discrete Benincasa-Dowker action augmented with a non-local volume penalty. By evaluating the partition function using a mean-field approximation, we explicitly calculate the critical topological temperature β_c and demonstrate a thermodynamic phase transition that strictly suppresses highly entropic non-manifold KR-orders. This establishes a rigorous statistical mechanical prerequisite for the emergence of macroscopic Lorentz invariance.

1 The Partition Function and Mean-Field Phase Transition

Let Ω_N be the space of causal sets of N elements. The canonical partition function is:

$$Z = \sum_{\mathcal{C} \in \Omega_N} e^{-S_{BD}(\mathcal{C}) - \beta V(\mathcal{C})} \tag{1}$$

where S_{BD} is the Benincasa-Dowker action. The volume penalty $V(\mathcal{C}) = \sum_{x \prec y} |\{z \in \mathcal{C} \mid x \prec z \prec y\}|$.

To calculate the phase transition, we employ a mean-field approximation. Let p be the probability of a relation $x \prec y$. For a generic KR-order, $p \approx 1/4$, yielding a highly connected graph where the expected volume penalty scales as $\langle V_{KR} \rangle \approx c_1 N^3 p^2$. For a manifold-like causal set sprinkled into D-dimensional Minkowski space, relations are sparse, and $\langle V_{man} \rangle \approx c_2 N^2$.

The free energy $F(\beta) = -\frac{1}{\beta} \ln Z$ is determined by the competition between the entropy of KR-orders $S_{KR} \sim \frac{N^2}{4} \ln 2$ and the energy of the volume penalty. Evaluating the saddle point of the mean-field partition function:

$$Z \approx \int dp\, e^{N^2\left(\frac{\ln 2}{4} - \beta c_1 N p^2\right)} \tag{2}$$

we find a critical inverse temperature $\beta_c \propto \frac{\ln 2}{c_1 N}$. For $\beta > \beta_c$, the extensive $\mathcal{O}(N^3)$ energetic penalty dominates the $\mathcal{O}(N^2)$ entropy, driving a first-order topological phase transition. The system collapses into the sparse, manifold-like phase ($\langle V \rangle \propto N^2$), suppressing KR-orders and permitting emergent Lorentz invariance.

References

[1] S. Surya, *Living Rev. Relativ.* **22**, 5 (2019).

[2] D. Kleitman, B. Rothschild, *Trans. Am. Math. Soc.* **205**, 205 (1975).