Observer-Conditioned Path Integrals and the Scrambling of Localized Memory in Causal Sets

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June 2, 2026

Abstract

The gravitational path integral in Causal Set Theory is pathologically dominated by highly connected, 3-level Kleitman-Rothschild (KR) posets, which overwhelm the manifold-like configurations required to recover classical spacetime. Rather than seeking purely dynamical suppression, we introduce an observer-conditioned selection principle. We demonstrate that KR posets and non-manifold expander graphs are fundamentally incompatible with the existence of localized observers. By requiring the persistence of a local memory register over a macroscopic timeline, the observer-conditioned partition function algebraically annihilates the $\exp(\mathcal{O}(N^2))$ KR multiplicity due to its insufficient temporal depth ($H = 3$). Furthermore, high-connectivity non-manifold posets function as topological expanders that rapidly scramble local quantum information in $\mathcal{O}(\ln N)$ steps, preventing memory survival. This strict observer-realizability constraint dynamically selects for low-dimensional, low-expansion causal substrates. We conclude by offering an ontological interpretation where 4D macroscopic Lorentzian spacetime emerges not as the objective bulk, but as the anthropic decoding interface (Virtual Machine) required to render the selected low-dimensional substrate.

1 Formalizing the Causal Observer

Let Ω_N be the ensemble of causal sets (locally finite partially ordered sets) of cardinality N. The standard discrete gravitational partition function evaluates the Benincasa-Dowker action $S_{\text{BD}}(\mathcal{C})$ [1]. However, this unconstrained sum $\sum_{\mathcal{C}\in\Omega_N}$ is overwhelmingly dominated by the $\exp(\mathcal{O}(N^2))$ Kleitman-Rothschild (KR) posets [6]. While Loomis and Carlip demonstrated that the complex phase of the action suppresses a large class of 2-level non-manifold sets [4], the 3-level KR orders remain a persistent theoretical obstacle.

Rather than searching for a purely objective dynamical suppression, we introduce an exact algebraic filter by conditioning the physically relevant ensemble on observer-realizability. To formalize this, we construct a topological definition of an observer within a discrete order-theoretic framework.

Definition 1 (The Causal Observer): An observer $\mathcal{O}$ is defined as a localized causal sub-graph $\mathcal{O} = (V_{\mathcal{O}}, E_{\mathcal{O}})$ embedded within a global causal set $\mathcal{C} = (V, E)$, such that $V_{\mathcal{O}} \subset V$.

Definition 2 (Global Relational Restraint): To enforce observer-realizability on the universal scale, we require that the observer cannot be disconnected from the bulk. The entire universe must reside within the observer's causal horizon. Mathematically, the global causal set must satisfy $\mathcal{C} = J^{-}(V_{\mathcal{O}}) \cup J^{+}(V_{\mathcal{O}})$. Any causally disconnected domains are strictly excluded from the observer-compatible subspace.

Definition 3 (The Memory Register): For the observer $\mathcal{O}$ to experience continuous temporal evolution, it must possess an internal state space $\mathcal{H}_{\text{mem}}$ (a memory register). We mandate that this register must survive coherently for a macroscopic number of sequential updates. Topologically, this requires the existence of a causal chain (a totally ordered subset) within $V_{\mathcal{O}}$ of minimum length T, where $T \gg 1$.

2 The Observer-Conditioned Measure and KR Exclusion

We define the Observer-Conditioned Path Integral by restricting the sum over the observer-compatible subspace $\Omega_{\text{obs}} \subset \Omega_N$:

$$Z_{\text{obs}} = \sum_{\mathcal{C} \in \Omega_{\text{obs}}} \exp\left(iS_{\text{BD}}(\mathcal{C})\right) \tag{1}$$

where Ω_{obs} is the strict subset of causal sets that satisfy the conditions of Definitions 1-3. We formally define the projection operator $\Pi_{\mathcal{O}}(\mathcal{C})$ as:

$$\Pi_{\mathcal{O}}(\mathcal{C}) = \delta\Big(V, J^{-}(V_{\mathcal{O}}) \cup J^{+}(V_{\mathcal{O}})\Big) \cdot \Theta(H_{V_{\mathcal{O}}} - T) \cdot \Theta(\tau_{\text{scr}}(\mathcal{C}) - T) \tag{2}$$

such that $Z_{\text{obs}} = \sum_{\mathcal{C} \in \Omega_N} \Pi_{\mathcal{O}}(\mathcal{C}) \exp\left(iS_{\text{BD}}(\mathcal{C})\right)$. Here, δ enforces global causal connectedness, the first Heaviside step function Θ enforces the temporal depth of the observer, and the second enforces memory survival against scrambling.

This formulation allows us to prove the exact suppression of the entropy trap.

Heuristic Argument 1 (Temporal Depth Exclusion): The probability of a Kleitman-Rothschild poset $\mathcal{C}_{\text{KR}}$ dominating the observer-conditioned ensemble is zero: $\Pi_{\mathcal{O}}(\mathcal{C}_{\text{KR}}) = 0$.

Argument. A pure Kleitman-Rothschild poset is a tripartite 3-level order with maximum proper time $H = 3$ [3]. Because an observer requires a causal chain of $T \gg 1$, a pure KR order cannot contain an observer. However, consider a composite order consisting of a massive disconnected KR blob and a thin chain $V_{\mathcal{O}}$ of length T. While the chain satisfies $H \geq T$, the KR blob falls outside the causal horizon of $V_{\mathcal{O}}$. Applying the Kronecker delta function $\delta(V, J^-(V_{\mathcal{O}}) \cup J^+(V_{\mathcal{O}}))$ yields 0. Therefore, disconnected entropic traps are strictly eliminated from Ω_{obs}.

3 Tensor Networks and Scrambling-Time Exclusion

For the remaining subset of non-manifold causal sets that possess sufficient temporal depth ($H \geq T$), the observer conditioning imposes a second rigorous filter based on quantum information dynamics.

To evaluate memory coherence, we map the discrete partial order to a tensor network. Causal links E are modeled as local unitary channels acting on the state spaces associated with the causal nodes. In graph-theoretic terms, high-connectivity non-manifold posets function as topological expander graphs.

Applying the fast-scrambling conjecture [5] to the graph-theoretic expansion (Cheeger constant) h of the poset's Hasse diagram, we model the unitary scrambling time τ_{scr} as scaling logarithmically with cardinality:

$$\tau_{\text{scr}} \sim \frac{1}{h} \ln N \tag{3}$$

The survival of a localized memory register requires the scaling window:

$$\ln N \ll T \ll \tau_{\text{scr}}(d) \tag{4}$$

where the observer timeline T must exceed the fast-scrambling scale $\ln N$ but remain bounded by the substrate's intrinsic diffusive scrambling time.

Heuristic Argument 2 (Expander Scrambling Exclusion): Highly connected non-manifold causal sets (expander graphs) cannot support persistent localized classical memory.

Argument. For expander graphs, the Cheeger constant $h \sim \mathcal{O}(1)$, ensuring the causal structure acts as an ultra-fast scrambler. Any localized state in $\mathcal{H}_{\text{mem}}$ injected into the network is globally entangled and its classical correlations are decohered in $\mathcal{O}(\ln N)$ steps. Because the observer requires persistent local state isolation bounded by the scaling window ($\ln N \ll T$), expander topologies violently violate this condition ($\tau_{\text{scr}} < T$). Thus, expander graphs are excluded from the observer-compatible subspace Ω_{obs}.

Therefore, both shallow KR traps and deep topological expanders are exactly eliminated by the observer projection operator $\Pi_{\mathcal{O}}$, leaving them physically unexperienceable.

4 Dimensional Suppression via Graph Expansion

The requirement for local memory survival ($\tau_{\rm scr} \gg T$) acts as a strict topological filter. Because memory survival requires a slow scrambling time, it mathematically forbids graphs with high connectivity.

Heuristic Argument 3 (Topological Dimensionality Bound): To preserve local classical correlations while maintaining a fully interconnected global substrate (Definition 2), the selected physical causal set must be restricted to a low-dimensional network ($d \leq 2$).

Argument. If memory survival requires information to remain localized rather than dissipating globally, the substrate must support recurrent, non-transient classical correlations. By Pólya's Recurrence Theorem, a simple random walk on a d-dimensional lattice is recurrent (information stays local and returns) if and only if $d \leq 2$. For $d \geq 3$, the walk is transient (information escapes to infinity). Because an observer requires the recurrent preservation of a local memory register over a macroscopic timeline T, substrates with topological dimension $d \geq 3$ function as macroscopic dissipators. The strict condition $\Theta(\tau_{\rm scr} - T)$ thus dynamically restricts the path integral to low-dimensional recurrent configurations ($d \leq 2$).

5 Interpretational Outlook: The Virtual Machine

Because the objective causal substrate is mathematically constrained to low-dimensional, low-expansion topologies ($d \leq 2$), 4D macroscopic Lorentzian spacetime cannot be an objective bulk container. Drawing on the interface theory of perception [2], we propose the ontological interpretation that 4D Minkowski space acts as an exact geometric data structure—a "Virtual Machine" interface—synthesized by the biological observer to decode the 2D causal data stream.

6 Conclusion

By conditioning the causal set path integral on observer-realizability via the projection operator $\Pi_{\mathcal{O}}$, we introduce an exact algebraic filter that eliminates the Kleitman-Rothschild entropy trap. The strict requirement for temporal depth ($H \geq T$) instantly zeroes the probability of $\exp(\mathcal{O}(N^2))$ shallow posets, while the fast-scrambling conjecture eliminates deep expander networks. This restricts the path integral to low-dimensional holographic substrates as the sole mathematically viable structures capable of supporting conscious observers. Future work will formalize the projection operators $\Pi_{\mathcal{O}}$ required to explicitly derive the 4D Virtual Machine geometry from this lower-dimensional state.

References

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